Government Debt Management and Inflation with Real and Nominal Bonds

Lukas Schmid[1] Vytautas Valaitis[2]

Alessandro T. Villa[3]

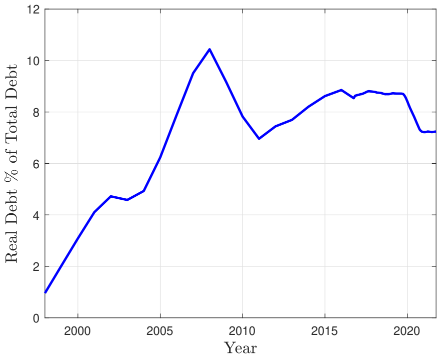
[1]USC and CEPR

[2]University of Surrey

[3]FRB Chicago

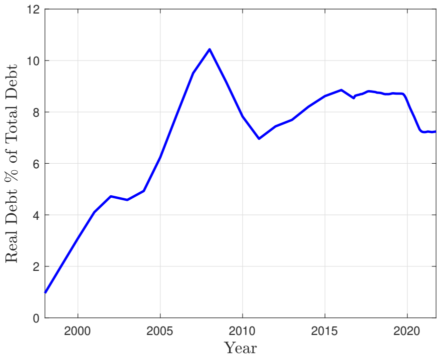
March 2024

TIPS?



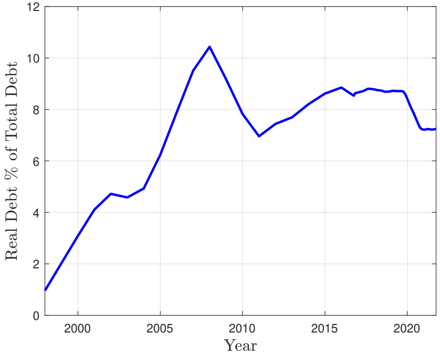
- The U.S. Treasury started issuing Treasury inflation-protected bonds (TIPS) in 1997

TIPS?



- The U.S. Treasury started issuing Treasury inflation-protected bonds (TIPS) in 1997
- How should governments optimally manage nominal and real bonds? Should governments issue more real bonds or less?

TIPS?



- The U.S. Treasury started issuing Treasury inflation-protected bonds (TIPS) in 1997
- How should governments optimally manage nominal and real bonds? Should governments issue more real bonds or less?
- **Does it matter?**

Does it Matter? - The U.K.

Gilts + Add to myFT

The UK issues more inflation-linked debt than anyone else. Has it worked?

Interest payments on 'linkers' to hit £92bn over the next 5 years

- The U.K. government started issuing inflation-linked gilts in 1981
- The share of indexed debt in the U.K. debt portfolio is around 25%

Does it Matter? - Canada

Bloomberg

● Live Now | **Markets** | Economics | Industries | Technology | Politics | Wealth | Pursuits | Opinion | Businessweek

Markets

Investors Ask Canada to Reverse Inflation-Linked Bond Freeze

- Demand for real-return bonds is too low, official said
- Decision is unrelated to high inflation, the official said

Bond market upsets aren't a typical feature of Canadian finances, but one revolt appears to have begun there recently. Buried in a 96-page economic update late last year, Canada's finance ministry led by Chrystia Freeland killed off its inflation-protected bond issuance programme — even as the country battles its worst price pressures for 40 years.

- The Canadian Government started issuing Real Return Bonds in 1991, but only two percent of issues were indexed

Does it Matter...

... in a high inflation - high debt environment?

Does it Matter...

... in a high inflation - high debt environment?

- US annual inflation had accelerated to ∼9.1% in 2022, the highest since 1982, and fell to around ∼5% in May 2023, and to ∼3.2% in November 2023

- Current US debt-gdp ratio is at around 123% and projected to rise to around 200% by 2050 (CBO)

Link

Does it Matter...

... in a high inflation - high debt environment?

- US annual inflation had accelerated to ∼9.1% in 2022, the highest since 1982, and fell to around ∼5% in May 2023, and to ∼3.2% in November 2023
  - Can governments use TIPS as part of their debt portfolio to commit to stable inflation rates?
- Current US debt-gdp ratio is at around 123% and projected to rise to around 200% by 2050 (CBO)
  - What is the government's incentive to monetize debt?

Link

What We Do

- We develop a DSGE model of fiscal and monetary policy where a government optimally manages debt with distortionary taxation and inflation concerns in a setting where
  - the government can issue long-term **non state-contingent** nominal and real (TIPS) bonds
  - inflation has real costs as prices are sticky

What We Do

- We develop a DSGE model of fiscal and monetary policy where a government optimally manages debt with distortionary taxation and inflation concerns in a setting where
  - the government can issue long-term **non state-contingent** nominal and real (TIPS) bonds
  - inflation has real costs as prices are sticky
- We characterize
  1. the optimal policy under full commitment (i.e. the Ramsey equilibrium)
  2. the optimal policy without commitment (i.e. the optimal time-consistent policy)

Basic Economic Forces

- **Nominal debt** can be inflated away, but bond prices reflect elevated inflation expectations

- **Real debt** prices are higher and more stable, but such debt constitutes a real commitment ex-post

What We Find

With **Full** Commitment (FC)

- The government exploits inflation fluctuations to create insurance

What We Find

With **Full** Commitment (FC)

- The government exploits inflation fluctuations to create insurance
  - borrow with the most volatile asset (nominal debt) and buy assets that pay in inflationary times (real assets)
  - optimal portfolio prescribes a large negative allocation to TIPS

What We Find

With **Full** Commitment (FC)

- The government exploits inflation fluctuations to create insurance
  - borrow with the most volatile asset (nominal debt) and buy assets that pay in inflationary times (real assets)
  - optimal portfolio prescribes a large negative allocation to TIPS

With **No** Commitment (NC)

- Future governments have an incentive to monetize debt ex-post

What We Find

With **Full** Commitment (FC)

- The government exploits inflation fluctuations to create insurance
  - borrow with the most volatile asset (nominal debt) and buy assets that pay in inflationary times (real assets)
  - optimal portfolio prescribes a large negative allocation to TIPS

With **No** Commitment (NC)

- Future governments have an incentive to monetize debt ex-post
- The current government
  - internalizes the future government's behavior through current elevated nominal bond prices
  - reduces the borrowing costs ex-ante substituting nominal debt with real debt

What We Find

With **Full** Commitment (FC)

- The government exploits inflation fluctuations to create insurance
  - borrow with the most volatile asset (nominal debt) and buy assets that pay in inflationary times (real assets)
  - optimal portfolio prescribes a large negative allocation to TIPS

With **No** Commitment (NC)

- Future governments have an incentive to monetize debt ex-post
- The current government
  - internalizes the future government's behavior through current elevated nominal bond prices
  - reduces the borrowing costs ex-ante substituting nominal debt with real debt
- With No Commitment, policies are quantitatively consistent with the US data.

What We Conclude

- Commitment frictions can rationalize observed real-nominal composition of government debt portfolio
- Commitment friction is quantitatively modest for the U.S. economy
- Framework with No Commitment appears as a good starting point for policy analysis

Related Literature

- Optimal policy under Full Commitment
  - *Lucas and Stokey, 1983*
  - With non-contingent real debt: *Aiyagari et al., 2002; Angeletos, 2002; Buera and Nicolini, 2004; Faraglia et al., 2019; Bhandari et al., 2017*
  - With non-contingent nominal debt: *Chari and Kehoe, 1999; Siu, 2004; Schmitt-Grohe and Uribe, 2004; Lustig et al., 2008; Marcet et al., 2013; Leeper and Zhou, 2021*.
  - With non contingent real and nominal debt: *Barro, 2006*
- Optimal policy with No Commitment
  - Markov-Perfect Fiscal Policy: *Klein, Krusell, and Rıos-Rull, 2008; Debortoli and Nunes, 2013; Debortoli, Nunes, and Yared, 2017; Clymo and Lanteri 2020*
  - With non contingent real and nominal debt: *Alvarez, Kehoe, and Neumeyer, 2004*
- Quantitative analysis of fiscal-monetary interactions
  - *Bianchi and Melosi (2017, 2019, 2022); Elenev, Landvoigt, Shultz, and Van Nieuwerburgh (2022); Corhay, Kind, Kung, and Morales (2023)*
- Role of TIPS
  - *Ang, Bekaert, and Wei (2008); Bekaert and Wang (2010); Pflueger and Viceira (2018)*

Model: Household

- Representative household with utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \cdot U(c_t, l_t).$$

- Budget constraint

$$c_t + Q_t B^h_{t+1} + q_t b^h_{t+1} = (1-\tau_t) w_t h_t + \frac{B^h_t}{\pi_t} + b^h_t$$

where B^h_t and b^h_t denote the household's nominal and real debt holdings, respectively

- Optimality Conditions

$$\begin{aligned} (1-\tau_t)\cdot u_c(c_t)\cdot w_t &= v_l(l_t), \\ u_c(c_t)\cdot Q_t &= \beta \mathbb{E}_t u_c(c_{t+1})\cdot \pi^{-1}_{t+1}, \\ u_c(c_t)\cdot q_t &= \beta \mathbb{E}_t u_c(c_{t+1}) \end{aligned}$$

Model: Firms

- An intermediate firm i (with production $Y_{i,t} = A \cdot h_{i,t}$) chooses prices and labor demand to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \mathcal{M}_{0,t} \cdot \left[\underbrace{P_{i,t} Y_{i,t} - P_t w_t h_{i,t} - P_t \Phi_t}_{\text{Dividend}} \right]$$

  where Φ_t is a Rotemberg quadratic adjustment cost with $\Phi_t = \frac{\varphi}{2}(\pi_t - \pi)^2$

- The intermediate goods demand is

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{1}{\nu}} Y_t$$

- The New Keynesian Phillips curve is

$$\frac{\nu - 1}{\nu} Y_t + \frac{Y_t}{\nu} \frac{w_t}{A} - \Phi'_t + \mathbb{E}_t[\mathcal{M}_{t,t+1} \cdot \Phi'_{t+1}] = 0$$

Technology, Government and Central bank

- The government budget is

$$q_t b^g_{t+1} + Q_t B^g_{t+1} + \tau_t w_t h_t = g_t + b^g_t + \frac{B^g_t}{\pi_t}$$

where B^g_t and b^g_t denote the government's nominal and real debt holdings, respectively

- The central bank applies a Taylor rule

$$\left(\mathbb{E}_t\left[\mathcal{M}_{t,t+1}\cdot\frac{1}{\pi_{t+1}}\right]\right)^{-1} = i_t = \frac{1}{\beta}\pi\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}$$

- The resource constraint is

$$c_t + g_t + \Phi_t = A\cdot h_t$$

Bond Market Clearing

Real and nominal bonds are in zero net supply

$$B_t^h + B_t^g = 0$$
$$b_t^h + b_t^g = 0$$

Bond Market Clearing

Real and nominal bonds are in zero net supply

$$\begin{aligned} B_t^h + B_t^g &= 0 \\ b_t^h + b_t^g &= 0 \end{aligned}$$

Convention

- A government's positive bond allocation (B^g / $b^g > 0$) comes with lending households
- A government's negative bond allocation (B^g / $b^g < 0$)comes with borrowing households

Implementability Constraints

- $s_t \equiv \tau_t A h_t w_t - g_t$ is government surplus

Implementability Constraints

- $s_t \equiv \tau_t A h_t w_t - g_t$ is government surplus
- Substitute away bond prices and taxes in the government budget consistently with household's optimality to get

$$\left(\frac{B_t}{\pi_t} + b_t\right) = s_t + \mathbb{E}_t\left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \left(\frac{B_{t+1}}{\pi_{t+1}} + b_{t+1}\right)\right],$$

and, iterating forward,

$$\frac{B_t}{\pi_t} + b_t = \mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j}\right]$$

Implementability Constraints

- $s_t \equiv \tau_t A h_t w_t - g_t$ is government surplus
- Substitute away bond prices and taxes in the government budget consistently with household's optimality to get

$$\left(\frac{B_t}{\pi_t} + b_t\right) = s_t + \mathbb{E}_t\left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \left(\frac{B_{t+1}}{\pi_{t+1}} + b_{t+1}\right)\right],$$

  and, iterating forward,

$$\frac{B_t}{\pi_t} + b_t = \mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j}\right]$$

- Higher debt requires more distortionary taxation going forward to balance the budget

Optimal Policy under Full Commitment

Ramsey Equilibrium

Given initial conditions, the *Ramsey planner* seeks stochastic sequences of policies $\pi(g^t), \tau(g^t), B(g^{t-1}), b(g^{t-1})$ and stochastic sequences of allocations $c(g^t), l(g^t)$ and prices $w(g^t)$ such that the household's time-0 expected life-time utility is maximized and such that

- the implementability constraint is satisfied,
- the New Keynesian Phillips curve holds, and
- the Taylor rule is satisfied

Ramsey Equilibrium: Optimality

Optimality conditions with respects to nominal and real bonds are

$$\begin{aligned}
\mu_t \cdot \mathbb{E}_t\left[\pi_{t+1}^{-1} \cdot u_c(c_{t+1})\right] &= \mathbb{E}_t\left[\mu_{t+1} \cdot u_c(c_{t+1}) \cdot \pi_{t+1}^{-1}\right], \\
\mu_t \cdot \mathbb{E}_t\left[u_c(c_{t+1})\right] &= \mathbb{E}_t\left[\mu_{t+1} \cdot u_c(c_{t+1})\right]
\end{aligned}$$

- The optimality consitions pin down dynamics for the recursive multipliers μ_t on the implementability constraints capturing the shadow value of relaxing the implementability constraint
- *Time-inconsistency*: If the government could renegotiate on its commitment, it would choose to ignore these multipliers in each period t

Key Force: Insurance

Outstanding liabilities at t...

$$\frac{B_{t-1}(s^{t-1})}{\pi_t(s^t)} + b_{t-1}(s^{t-1}) = \tilde{b}_t(s^t)$$

...are measurable wrt s^t!

Can we exploit fluctuations in inflation to complete the market with real and nominal bonds with the same maturity?

- If we have as many bonds with non perfectly correlated returns as realization of the exogenous state, then yes

Mechanism: One-Period Model

- Time is $t = 0, 1$
- $u(c) = c$ and $v(h) = h^2/4$
- Two realizations of exogenous shocks: (π^L, g^L) and (π^H, g^H)
- Initial conditions: B_0, b_0, g_0, π_0

Mechanism: One-Period Model

- Time is $t = 0, 1$
- $u(c) = c$ and $v(h) = h^2/4$
- Two realizations of exogenous shocks: (π^L, g^L) and (π^H, g^H)
- Initial conditions: B_0, b_0, g_0, π_0

Optimal nominal and real bond choices:

$$\begin{aligned} \mu_0 \cdot \mathbb{E}_0\left[\frac{1}{\pi_1}\right] &= \mathbb{E}_0\left[\mu_1 \cdot \frac{1}{\pi_1}\right], \\ \mu_0 &= \mathbb{E}_0[\mu_1] \end{aligned}$$

- μ_0 and μ_1 are Lagrange multipliers on the implementability constraints
- larger multipliers imply higher needs to resort to distortionary taxation ('excess burden of government debt')

Mechanism: One-Period Model

Debt Management, Labor and Tax Smoothing. Given initial conditions B_0, b_0, g_0, π_0, optimal nominal and real debt management and tax management are such that smoothing of taxes and leisure is achieved *across states*

$$l_1^H = l_1^L \iff \tau_1^H = \tau_1^L, \tag{1}$$

where l_1^L and l_1^H denote leisure at time 1 in the low and high state, respectively.

Mechanism: One-Period Model

Debt Management, Labor and Tax Smoothing. Given initial conditions B_0, b_0, g_0, π_0, optimal nominal and real debt management and tax management are such that smoothing of taxes and leisure is achieved *across states*

$$l_1^H = l_1^L \iff \tau_1^H = \tau_1^L, \tag{1}$$

where l_1^L and l_1^H denote leisure at time 1 in the low and high state, respectively.

Smoothing of taxes and leisure is achieved *across time*

$$l_1^x = l_0 \iff \tau_1^x = \tau_1^0, \tag{2}$$

where $x \in \{L, H\}$.

Mechanism: One-Period Model

Optimal Nominal and Real Debt Management. Given the initial conditions, optimal nominal debt management is such that

$$B_1^* = \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H,$$

satisfies the intra-temporal (cross-states) smoothing condition (1).

Mechanism: One-Period Model

Optimal Nominal and Real Debt Management. Given the initial conditions, optimal nominal debt management is such that

$$B_1^* = \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H,$$

satisfies the intra-temporal (cross-states) smoothing condition (1).

Optimal real debt management is such that

$$b_1^* = \frac{1}{1+\beta}\left[\frac{B_0}{\pi_0} + b_0 - \left(\frac{1}{\pi_0} + \beta \mathbb{E}_0\left[\frac{1}{\pi_1}\right]\right) B_1^*\right],$$

satisfies the inter-temporal smoothing condition (2)

Mechanism: One-Period Model

Optimal Nominal and Real Debt Management. Given the initial conditions, optimal nominal debt management is such that

$$B_1^* = \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H,$$

satisfies the intra-temporal (cross-states) smoothing condition (1).

Optimal real debt management is such that

$$b_1^* = \frac{1}{1+\beta}\left[\frac{B_0}{\pi_0} + b_0 - \left(\frac{1}{\pi_0} + \beta \mathbb{E}_0\left[\frac{1}{\pi_1}\right]\right) B_1^*\right],$$

satisfies the inter-temporal smoothing condition (2)

- If expenditures are inflationary, $\pi_H > \pi_L$, real debt is negative,
- If expenditures are deflationary, $\pi_H < \pi_L$, *nominal* debt is negative: nominal assets appreciate when financing needs are high.

Quantitative Analysis: Recursive Solution

Ramsey Problem with incomplete markets and bonds with $N = 5$...

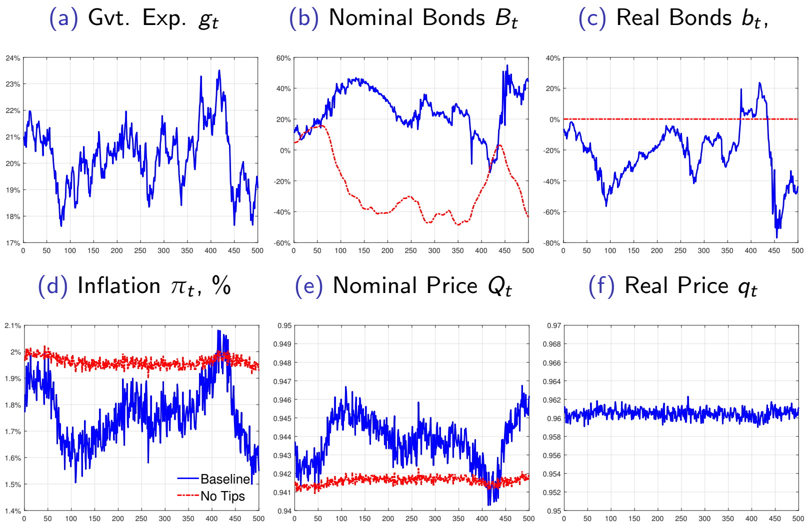
$$\mathcal{I}_t = \{g_t, \{B^N_{t-k}\}^N_{k=1}, \{b^N_{t-k}\}^N_{k=1}, \{\mu_{t-k}\}^N_{k=1}, \{\lambda^T_{t-k}\}^N_{k=1}, \{\lambda^\pi_{t-k}\}^N_{k=1}\}$$

...requires to solve for 10 policy functions of 26 state variables.

- We use a stochastic simulation approach...
  - den Haan and Marcet (1990), Faraglia et al. (2019), Judd et al. (2011).
- ...combined with machine learning.
  - Duarte (2018), Azimovich et al.(2019), Maliar et al.(2021).

Algorithm

Equilibrium Path: Leveraged Position and Rebalancing



- Blue - Baseline model. Red - Model without TIPS.

Extensions

Full Commitment results robust to multiple model extensions:

- Maturity
  - Spread inflationary distortion over longer periods, even more leveraged positions. Link
- Slope of the NKPC
  - A flatter NKPC implies even more leveraged positions. Link
- Monetary Policy Tightness
  - Tighter monetary policy implies less volatile inflation and, therefore, more leveraged bond portfolio. Link
- TFP shocks still yields the same portfolio composition
  - TFP shocks are deflationary but the correlation between the net present value of surpluses and inflation is what matters. Link

Taking Stock

Under Full-Commitment:

- Government borrows in nominal bonds and accumulates real assets,

Taking Stock

Under Full-Commitment:

- Government borrows in nominal bonds and accumulates real assets,
- Active use of inflation for hedging purposes,

Taking Stock

Under Full-Commitment:

- Government borrows in nominal bonds and accumulates real assets,
- Active use of inflation for hedging purposes,
- Portfolio rebalancing over the cycle,

Taking Stock

Under Full-Commitment:

- Government borrows in nominal bonds and accumulates real assets,
- Active use of inflation for hedging purposes,
- Portfolio rebalancing over the cycle,
- When lending not allowed, only issue nominal bonds. Link

Taking Stock

Under Full-Commitment:

- Government borrows in nominal bonds and accumulates real assets,
- Active use of inflation for hedging purposes,
- Portfolio rebalancing over the cycle,
- When lending not allowed, only issue nominal bonds. Link

What could explain the issuance of real debt?

Optimal Policy without Commitment

Symmetric Markov-Perfect Equilibrium

- We view the public sector as a succession of decision makers with no commitment to future realized policies.

Symmetric Markov-Perfect Equilibrium

- We view the public sector as a succession of decision makers with no commitment to future realized policies.
- The current government at t seeks (B', b', τ) to strategically best respond to the future government.

Symmetric Markov-Perfect Equilibrium

- We view the public sector as a succession of decision makers with no commitment to future realized policies.
- The current government at t seeks (B', b', τ) to strategically best respond to the future government.
- We focus on the Symmetric Markov-Perfect Equilibrium of the associated infinite-horizon game.

Symmetric Markov-Perfect Equilibrium

- We view the public sector as a succession of decision makers with no commitment to future realized policies.
- The current government at t seeks (B', b', τ) to strategically best respond to the future government.
- We focus on the Symmetric Markov-Perfect Equilibrium of the associated infinite-horizon game.
- Consider the interplay between *market incompleteness* and the *no commitment* frictions.

Mechanism: Two-Date Model

- $t = 0, 1$.
- Gov. $t = 1$ chooses π_1.
- Gov. $t = 0$ chooses B_1 and b_1.
- $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $v(l) = \frac{l^{1-\eta_l}}{1-\eta_l}$.
- $\Phi = \frac{\varphi}{2}(\pi_t - \pi)^2$.

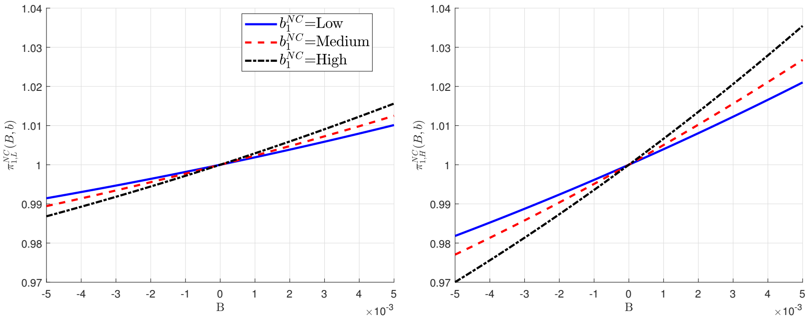
Mechanism: Two-Date Model

- $t = 0, 1$.
- Gov. $t = 1$ chooses π_1.
- Gov. $t = 0$ chooses B_1 and b_1.
- $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $v(l) = \frac{l^{1-\eta_l}}{1-\eta_l}$.
- $\Phi = \frac{\varphi}{2}(\pi_t - \pi)^2$.

Optimal Inflation at $t = 1$:

$$-u_c(c_1)\Phi_\pi(\pi_1) = \mu_1 \left(\frac{B_1}{\pi_1^2} + h_1 \frac{\partial \tau_1}{\partial \pi_1} \right).$$

Mechanism: Date Two Inflation



(a) Low g_1 (b) High g_1

Date Two inflation:

- Increases in nominal debt.
- More sensitive to debt levels when government expenditure is high.

Mechanism: Date One Debt

Planner at $t = 0$ internalizes the effect of higher B_1 on current prices through coupled Generalized Euler Equations:

$$\begin{aligned}
\mu_0 \left(Q + \frac{\partial Q}{\partial B_1} B_1 + \frac{\partial q}{\partial B_1} b_1 \right) &= \beta \mathbb{E}_0 \left[\frac{\mu_1}{\pi_1} \right], \\
\mu_0 \left(q + \frac{\partial Q}{\partial b_1} B_1 + \frac{\partial q}{\partial b_1} b_1 \right) &= \beta \mathbb{E}_0 [\mu_1].
\end{aligned}$$

Mechanism: Date One Debt

Planner at $t = 0$ internalizes the effect of higher B_1 on current prices through coupled Generalized Euler Equations:

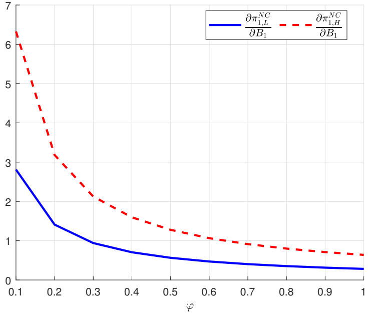
$$\mu_0 \left(Q + \frac{\partial Q}{\partial B_1} B_1 + \frac{\partial q}{\partial B_1} b_1 \right) = \beta \mathbb{E}_0 \left[\frac{\mu_1}{\pi_1} \right],$$

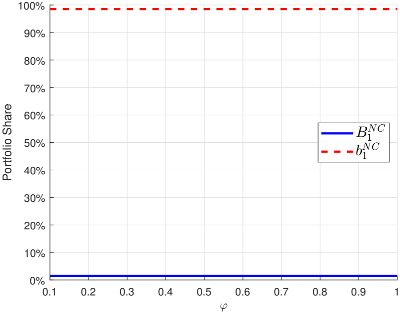
$$\mu_0 \left(q + \frac{\partial Q}{\partial b_1} B_1 + \frac{\partial q}{\partial b_1} b_1 \right) = \beta \mathbb{E}_0 [\mu_1].$$

Expanding the strategic bias term:

$$\frac{\partial Q}{\partial B_1} = \frac{\beta}{u_c(c_0)} \mathbb{E}_0 \left[\frac{u_{cc}(c_1)}{\pi_1} - \frac{u_c(c_1)}{\pi_1^2} \frac{\partial \pi_1}{\partial B_1} \right]. \tag{3}$$

Mechanism: Date One Debt





Date One policy

- Inflationary bias state dependent, - like depends on cost of inflation and on the level of g.
- Predominantly issue real debt.

Sketch of Symmetric MPE

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right],$$

Sketch of Symmetric MPE

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right],$$

Symmetric Markov-perfect equilibrium with $x \equiv (B, b, g)$. All future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, $\tilde{g}(x)$, and $\tilde{\pi}(x)$.

Sketch of Symmetric MPE

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right],$$

Symmetric Markov-perfect equilibrium with $x \equiv (B, b, g)$. All future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, $\tilde{g}(x)$, and $\tilde{\pi}(x)$.

Let $\tilde{W}(x)$ be the NPV of government utility associated with these policies. The government in power at time t chooses allocations and wage (c, h, w), as well as policies (B', b', τ, π) to maximize

$$u(c) + v(l) + \beta \mathbb{E} \tilde{W}(x'),$$

subject to private sector constraint, budget constraints, NKPC and the Taylor rule.

Sketch of Symmetric MPE

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right],$$

Symmetric Markov-perfect equilibrium with $x \equiv (B, b, g)$. All future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, $\tilde{g}(x)$, and $\tilde{\pi}(x)$.

Let $\tilde{W}(x)$ be the NPV of government utility associated with these policies. The government in power at time t chooses allocations and wage (c, h, w), as well as policies (B', b', τ, π) to maximize

$$u(c) + v(l) + \beta \mathbb{E} \tilde{W}(x'),$$

subject to private sector constraint, budget constraints, NKPC and the Taylor rule.

- We solve for the symmetric MPE using an algorithm similar in spirit to Clymo and Lanteri, 2020.

Quantitative Results: Commitment Dominates Hedging

- Each gov. chooses (B', b', τ, π)
- We solve for the symmetric MPE

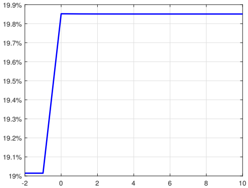
Quantitative Results: Commitment Dominates Hedging

- Each gov. chooses (B', b', τ, π)
- We solve for the symmetric MPE

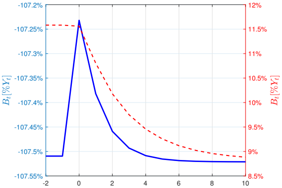
Description	Moments	NC No TIPS	NC Baseline
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.6	1.9
Avg. Tax [%]	$\mathbb{E}(\tau)$	23.6	21.4
Avg. Short Nom. Rate [%]	$\mathbb{E}(i)$	5.7	6.1
Avg. Real to GDP	$\mathbb{E}(b/Y)$	-	0.64
Avg. Nominal to GDP	$\mathbb{E}(B/Y)$	0.07	-1.08
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.991	0.995
Corr. Tax and GDP	$\rho(\tau, Y)$	0.952	0.971
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.656	0.15
Corr. Inflation and Real	$\rho(\pi, b)$	-	0.295
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.48	0.1

Conditional Dynamics: under NC Real Debt Stabilizes Inflation

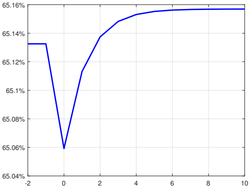
(a) Gvt. Exp. g_t [% GDP]



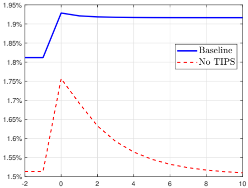
(b) Nominal Bonds B_t [% GDP]



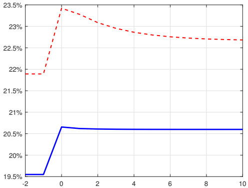
(c) Real Bonds b_t [% GDP]



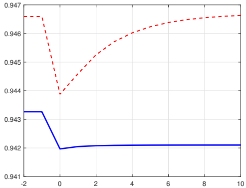
(d) Inflation $\pi_t - 1$ [%]



(e) Tax rate τ_t [%]



(f) Nominal Price Q_t



- Blue - Baseline model. Red - Model without TIPS.

Quantitative Results: Commitment Friction versus Inflation Costs

- Slope of the NKPC
- Hawkishness of the Monetary Authority
- Extension with endogenous g shows inflation is less correlated with y and B, consistent with the U.S. data

Quantitative Results: Commitment Friction versus Inflation Costs

- Slope of the NKPC
- Hawkishness of the Monetary Authority
- Extension with endogenous g shows inflation is less correlated with y and B, consistent with the U.S. data

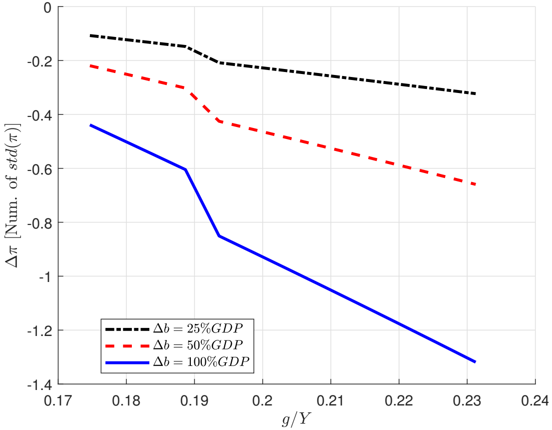
Moments	Model					Data/Target
	$\phi_\pi = 1.2$ $\varphi = 20$ (a)	$\phi_\pi = 1.22$ $\varphi = 20$ (b)	$\phi_\pi = 1.25$ $\varphi = 20$ (c)	$\phi_\pi = 1.2$ $\varphi = 22.5$ (d)	$\phi_\pi = 1.2$ $\varphi = 25$ (e)	
$\mathbb{E}(\pi) - 1$	1.89	1.89	1.88	1.9	1.9	2
$\mathbb{E}(\tau)$	24.1	24.2	24.1	24.1	24.1	22.8
$\mathbb{E}(b/(b+B))$	0.18	0.05	0.04	0.09	0.02	0.07
$\mathbb{E}(B/(b+B))$	0.82	0.95	0.96	0.91	0.98	0.93
$\rho_1(b/(b+B))$	0.948	0.944	0.855	0.939	0.878	0.94
$\rho_1(b/Y)$	0.949	0.947	0.866	0.941	0.871	0.995
$\rho(g, Y)$	0.999	0.996	0.963	0.997	0.99	0.23
$\rho(\tau, Y)$	0.999	0.979	0.814	0.981	0.939	0.35
$\rho(\pi, Y)$	0.943	0.719	-0.314	0.765	0.492	0.06
$\rho(\pi, b)$	-0.412	-0.384	-0.776	-0.422	-0.475	0.47
$\rho(\pi, B)$	0.412	0.444	0.863	0.458	0.584	-0.07

Can more real debt help to lower inflation?

Rebalance tomorrow's debt: $\Delta\pi_t = \frac{\partial \pi_t}{\partial b_{t+1}} \cdot \Delta b_{t+1} + \frac{\partial \pi_t}{\partial B_{t+1}} \cdot \Delta B_{t+1}$

Can more real debt help to lower inflation?

Rebalance tomorrow's debt: $\Delta \pi_t = \frac{\partial \pi_t}{\partial b_{t+1}} \cdot \Delta b_{t+1} + \frac{\partial \pi_t}{\partial B_{t+1}} \cdot \Delta B_{t+1}$



Conclusions

- Under full commitment
  - Leveraged portfolio of positive nominal and negative real debt.
  - Rebalancing towards real debt in inflationary times.

Conclusions

- Under full commitment
  - Leveraged portfolio of positive nominal and negative real debt.
  - Rebalancing towards real debt in inflationary times.
- Under no commitment
  - The future gov. has an incentive to monetize debt ex-post.
  - The current gov.
    - Internalizes the future gov.'s behavior through the current nominal price, which reflects the elevated inflation expectations.
    - Reduces the borrowing costs ex-ante substituting nominal debt with real debt.

Conclusions

- Under full commitment
  - Leveraged portfolio of positive nominal and negative real debt.
  - Rebalancing towards real debt in inflationary times.
- Under no commitment
  - The future gov. has an incentive to monetize debt ex-post.
  - The current gov.
    - Internalizes the future gov.'s behavior through the current nominal price, which reflects the elevated inflation expectations.
    - Reduces the borrowing costs ex-ante substituting nominal debt with real debt.
  - Rationalize positive real debt observed in the US. Good starting point for policy analysis.

Thank You!

Parameter Values

Parameter	Value	Description
β	0.96	Discount factor
γ	2	Relative risk aversion
η	1.8	Leisure utility parameter
A	1.0	Technology level
χ	4.3276	Labor utility parameter
$-\frac{1}{\nu}$	-10	Price elasticity of demand
φ	20	Rotemberg adj. cost, in line with Clarida, Gali, Gertler (1999)
ϕ_π	1.2	Taylor rule response to inflation
Π	1.02	SS inflation, Fed target
ρ, σ_ϵ	0.977, 0.0161	g_t Persistence and std, BEA
$\mu(1-\rho)$	0.2	Ratio of gvt. expenditure to GDP, BEA
N	1	Maturity of gvt. debt

Quantitative Analysis

Bonds Optimality

The first order condition with respect to nominal bonds is

$$\mu_t = \left[\mathbb{E}_t[U_{1,t+N}/\Pi_{j=1}^N \pi_{t+j}]\right]^{-1}\left[\mathbb{E}_t[\mu_{t+1}U_{1,t+N}/\Pi_{j=1}^N \pi_{t+j}] + \frac{\xi_{U,t}}{\beta^N} - \frac{\xi_{L,t}}{\beta^N}\right]$$

where $\xi_{U,t}$ and $\xi_{L,t}$ are the Lagrange multipliers on the upper and lower bounds, respectively.

The first order condition with respect to real bonds is

$$\mu_t = \left[\mathbb{E}_t\left[U_{1,t+N}\right]\right]^{-1}\left[\mathbb{E}_t[\mu_{t+1}U_{1,t+N}] + \frac{\xi^T_{U,t}}{\beta^N} - \frac{\xi^T_{L,t}}{\beta^N}\right]$$

Algorithm

Algorithm: Part II

- When equilibrium sequences are generated through stochastic simulation, the simulated states are likely multicollinear. Go

Algorithm: Part II

- When equilibrium sequences are generated through stochastic simulation, the simulated states are likely multicollinear. Go
- Given a guess for the policy functions and initial states, a typical stochastic simulation algorithm would require the following steps.

Algorithm: Part II

- When equilibrium sequences are generated through stochastic simulation, the simulated states are likely multicollinear. Go
- Given a guess for the policy functions and initial states, a typical stochastic simulation algorithm would require the following steps.

1. Start with a core set of state variables (a subset of $\mathcal{I}_t$).
    - Solve all optimality conditions on a time horizon T.
    - Use the simulated data to update the policy functions.
    - Iterate till convergence.

Algorithm: Part II

- When equilibrium sequences are generated through stochastic simulation, the simulated states are likely multicollinear. Go
- Given a guess for the policy functions and initial states, a typical stochastic simulation algorithm would require the following steps.

1. Start with a core set of state variables (a subset of $\mathcal{I}_t$).
   - Solve all optimality conditions on a time horizon T.
   - Use the simulated data to update the policy functions.
   - Iterate till convergence.
2. Doesn't work? Use residuals (orthogonal) to update core set.

Algorithm

Algorithm: Part III

- We represent the policy functions with a neural network and we use a stochastic gradient descent algorithm to train it.

Algorithm: Part III

- We represent the policy functions with a neural network and we use a stochastic gradient descent algorithm to train it.
- The network proved robust to multi-collinearity and allowed to remove the loop required to look for a core set of states.

Algorithm: Part III

- We represent the policy functions with a neural network and we use a stochastic gradient descent algorithm to train it.
- The network proved robust to multi-collinearity and allowed to remove the loop required to look for a core set of states.

1. Start with a core set of state variables (a subset of $\mathcal{I}_t$).
   - Solve all optimality conditions on a time horizon T.
   - Use the simulated data to update the policy functions.
   - Iterate till convergence.
2. Doesn't work? Use residuals (orthogonal) to update core set.

Algorithm: Part III

- We represent the policy functions with a neural network and we use a stochastic gradient descent algorithm to train it.
- The network proved robust to multi-collinearity and allowed to remove the loop required to look for a core set of states.

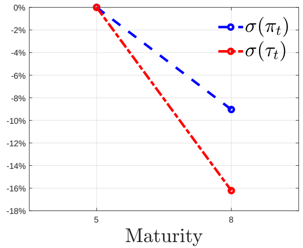
1. Start with a core set of state variables (a subset of $\mathcal{I}_t$).
   - Solve all optimality conditions on a time horizon T.
   - Use the simulated data to update the policy functions.
   - Iterate till convergence.
2. Doesn't work? Use residuals (orthogonal) to update core set.

This reduces the complexity of the algorithm significantly since the maximum number of combinations are $\sum_{k=2}^{N} C_{N,k}$. We gain in speed and scalability.

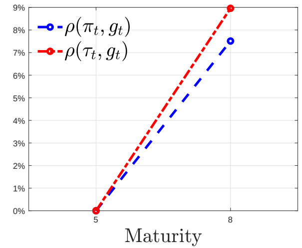
⏩ Algorithm

Extensions: ↑ Maturity → ↓ Inflation Volatility

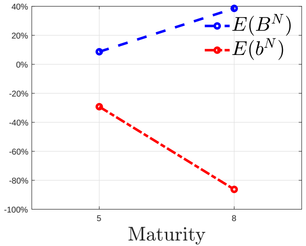
(a) Policy Volatility



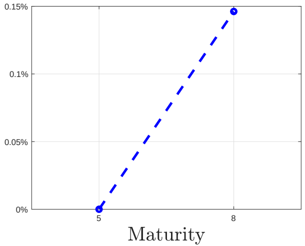
(b) Policy Correlation



(c) Nominal and Real Debt



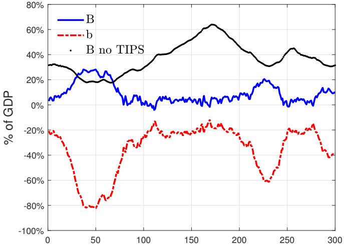
(d) Welfare Consumption Equivalent



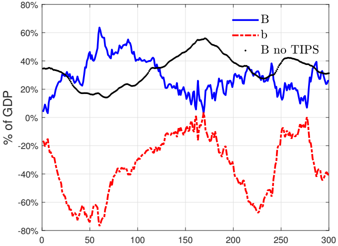
Back

Extensions: Slope of the Philips Curve

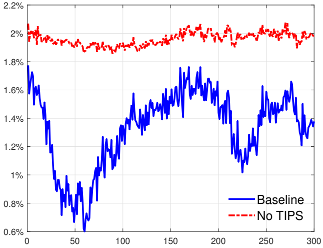
(a) Bonds



(b) Bonds

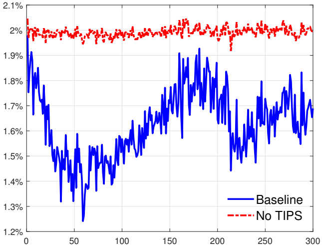


(c) Inflation π_t [%]



Calibration

(d) Inflation π_t [%]



High Adjustment Cost

Extensions: Alternative Shocks

Table: Comparison with a model with TFP shocks

Description	Moments	No TIPS *g shocks*	Baseline *g shocks*	Baseline *TFP shocks*
Avg. Real to GDP	$\mathbb{E}(b^N/Y)$	-	**-0.28**	**-0.37**
Avg. Nominal to GDP	$\mathbb{E}(B^N/Y)$	0.40	**0.24**	**0.40**
Corr. Tax and GDP	$\rho(\tau, Y)$	0.54	0.3	-0.84
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.39	0.39	-0.66
Corr. Tax and Inflation	$\rho(\tau, \pi)$	0.84	0.96	0.81
Corr. Inflation and Real	$\rho(\pi, b^N)$	-	**0.93**	**0.45**
Corr. Inflation and Nominal	$\rho(\pi, B^N)$	0.68	**-0.69**	**-0.22**
Corr. Real and Nominal	$\rho(b^N, B^N)$	-	-0.84	-0.70

Back

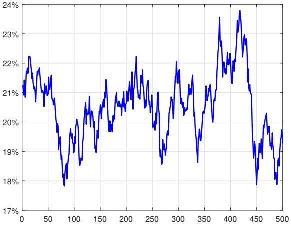
Extensions: Monetary Policy Tightness

	$\rho(b_t^N, B_t^N)$	$\rho(B_t^N - b_t^N, g_t)$	$\mathbb{E}(B_t^N / Y_t)$	$\mathbb{E}(b_t^N / Y_t)$	$\sigma(\pi_t)$
$\phi_\pi = 1.2$	-0.8545	-0.8046	0.0912	-0.3054	0.0040
$\phi_\pi = 1.25$	-0.9171	-0.8332	0.4972	-0.2780	0.0032

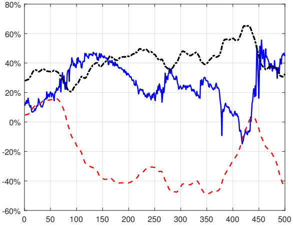
Back

No Lending

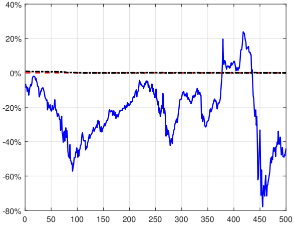
(a) Gvt. Exp. g_t



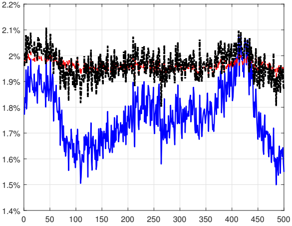
(b) Nominal Bonds B_t



(c) Real Bonds b_t



(d) Inflation π_t, %



- Blue - Baseline model, Red - Model without TIPS, Black - No Lending. ⏩ Back

Planners Problem

$$\max \; c_0 - \frac{h_0^2}{2} + \beta E_0 (c_1 - \frac{h_1^2}{2})$$

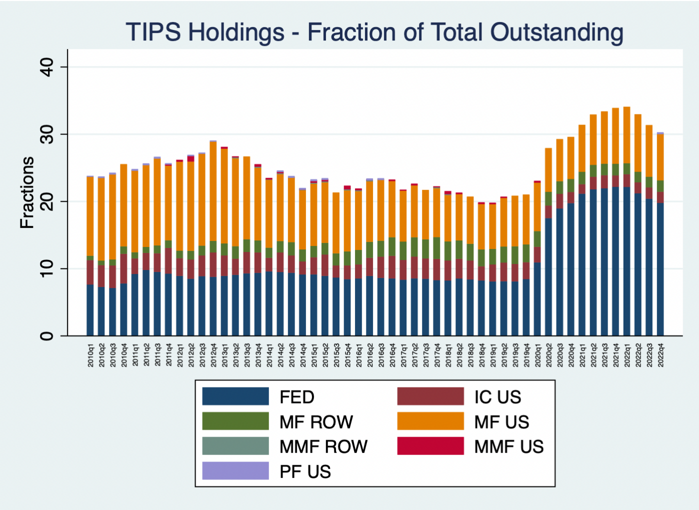
$$s.t.$$

$$\frac{B_0}{\pi_0} + b_0 + g_0 = h_0 \left(1 - \frac{h_0}{2}\right) + \beta E_0[\pi_1^{-1}] B_1 + \beta b_1,$$

$$\frac{B_1}{\pi_1} + b_1 + g_1 = h_1 \left(1 - \frac{h_1}{2}\right)$$

and resource constraint Back

TIPS Holdings



▶▶ Back

Source: Jansen, Li, Schmid (202?)